Based on version 2.2, last update : 20/07/2010

Statistics Module

The Statistics module in Win-Tensor allows to characterize the distribution of orientation data and to separate them into subset. The distribution analysis discriminates between cluster and girdle distributions and provides the average orientation and for the fault-slip data in the case of cluster distribution and the pole of the best-fit great circle in the case of girdle distributions. The subset separation is based on the procedure for separation of heterogeneous sets of orientation data into subsets developed by Huang and Charlesworth (1989). In this module, planar data (defined by their pole) and linear data are treated separately. The concentration parameters are also provided: the normalized length of the resultant vector R and the confidence cone angle.

This module is illustrated with the help of file GirdleCluster.wtd composed on planar data only, grouped into 4 subsets with different kind of distribution: a girdle distribution (29 data), a cluster distribution (29) data an homogeneous distribution (17 data) and a strongly clustered set (20 data). This data set contains no linear data, but the distribution analysis and separation process is similar as for planar data. The weighting of all data has been set to 2 so activating / deactivating the weighting function will have no effect.
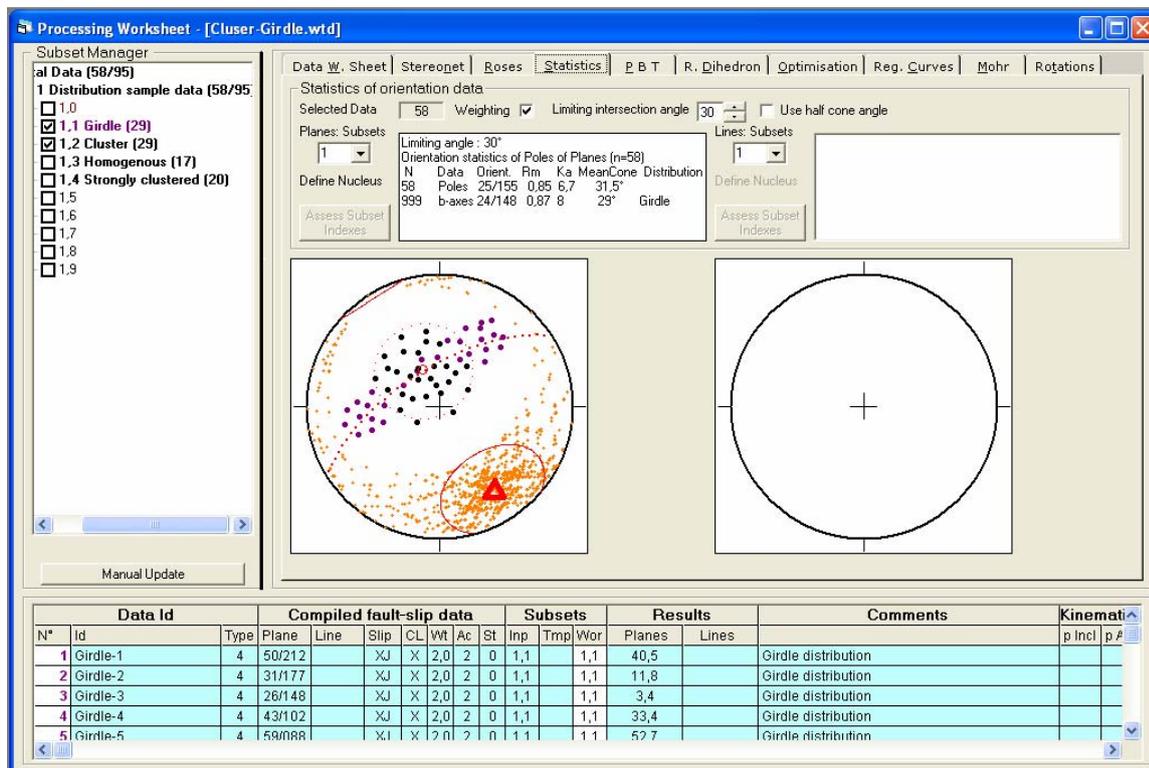

Figure X.1a: Statistics pane of the Processing Sheet, with orientation statistics for the selected data grouped into a single data set.

The Statistics pane (Fig. X.1a) is composed of fist line indicating the number of selected data and allowing to control the weighting option, and the value of the limiting intersection angle.

Under this first line, two similar groups of controls appear the left one for the orientation of planar data and the right one for linear data. Each of these groups is composed of a drop-down list defining the number of subsets, a text area displaying the analytical results, a picture area displaying a stereogram and a button "Assess Subset Indexes" which is initially disabled.

When only one subset is defined in the drop-down list, the Statistics module characterizes the distribution of the selected orientation data (Fig. X1a). When several subsets are defined (max. 5), it allows to separate the initial data sets into homogeneous subsets (Fig. X.1b).

1/ Distribution of orientation data

Statistics of clustered orientation data

The average orientation and statistical dispersion (or concentration) of linear orientation data are computed by vectorial summation of their direction cosines, taking into account the weighting factor associated to each data. In this process, the poles to planes and lines are considered as vector. As vectors are defined by their direction, the vectors are flipped to point in the same direction of the principal axis before their summation, there fore considering parallel vectors in opposing direction as similar orientations as in the Watson bipolar distribution (Fisher et al., 1987). The implementation of this statistics in Win-Tensor has been inspired by the published programs Watson (Dzik, 1992) and Orient (Charlesworth et al., 1989).

The principal axis orientation of the resulting vector represents the average orientation of the orientation data set. The length (or magnitude) of the unit vectors are multiplied by the weight of the corresponding orientation data before they are summed up into a resulting vector R. The length of the latter is divided by the sum of all weighing factors (or the number of data if the weighting is not applied), leading a mean resulting vector Rm. The length (or magnitude) of Rm expresses the degree of clustering of the vectors. It ranges from a maximum of 1 for perfectly clustered populations where all orientation data are strictly parallel and it decreases progressively with data dispersion to to ~0.5 for uniformly distributed data (perfectly spread) on the sphere.

The Ficher concentration parameter KA = (n – 2) / n - R), where R is the length of the resultant vector, ranges from zero for uniformly distributed data on the sphere and to infinity for a perfectly clustered data set (Huang and Charlersworth, 1989; Charlersworth et al., 1989).

The mean distance between individual vectors and the mean vector (Mean Cone Angle) is defined by the half-apex angle MCA = arc cosine (Rm) of the mean circular cone around the principal axis direction. The mean cone angle is meaningful for valid cluster (Fisher) distributions, but is no more valid for point distributions that are not simple cluster.

The analytical results are displayed on the text area and illustrated on the stereographic graphic display (Example X.1). The graphic options are controlled by the Graphic Options windows that can be opened by a right click on the picture or from the Menu/Graphics/Graphic Option. The content of the text box can be highlighted and copied to the clipboard or displayed on a separate window using the command Menu/Edit/Display

Results. The content can be copied to the clipboard by a right click and selecting option Copy Graphic to Clipboard on the floating menu.

Note: The direction cosines are the three component of a unit vector obtained from the orientation parameters (azimuth and plunge of a pole to plane or of a line) as follows:

X = sin(azimuth) * cos (plunge)
Y = cos(azimuth) * cos(plunge)
Z = -sin(plunge)

Girdle distributions

An alternative way to analyze the distribution of orientation data is to determine eigenvectors and eigenvalues from direction cosines matrix instead of using vectorial data and to evaluate the shape of the distribution (cluster v/s girdle) in a modified Flynn plot.

Instead, we developed a convenient and straightforward method for analyzing data distributed along a great circle on a hemispheric projection.

Cylindrical folds where the normal to the bedding planes measured unevenly on the cylindrically folded surface all lie in a single plane and plot on a stereographic projection along a best-fit great circle, the orientation of the fold axis corresponding to the pole of that plane (e.g. Lisle and Leyshon, 2004). In addition, the intersection between any pairs of bedding planes lying on the cylindrically folded surface (b-axes) also plot at the vicinity of the fold axis. By extension and considering that a perfect girdle distribution of orientation poles defines a plane whose pole is sub-parallel to the b-axis of any pairs of intersecting planes, a rapid procedure has been developed to statistically describe girdle distributions.

It involves first the calculation of the orientation of the b-axes of all possible pairs of planes in the data set. The number of intersections $nI = (n^2 - n)/2$ with n = the number of data in the set. The cluster distribution of the b-axes is computed as above, with the mean orientation representing the pole of the great circle best fitting the girdle distribution of poles. The length of the mean resulting vector summing all the b-axes (Rm_b-axes) is an expression of the degree of clustering of the intersection axes, as the derived KA concentration parameter and confidence cone angle. These dispersion parameters are good estimates of the degree of fitting of the great circle to the girdle distribution. They can be compared for the same data set to the dispersion parameters of the cluster distributions, with (Rm_poles) as the length of the mean resulting vector summing all the poles to the planes.

The shape of the distribution is determined by comparing the dispersion parameters of both cluster and girdle distributions for the same data set. A distribution is considered as Clusters when their distribution of Poles have longer Rm axes and lower Ka and confidence cone angles than the corresponding distribution of b-axes. Conversely, Girdle distributions have longer Rm axes and lower Ka and confidence cone angles for the b-axis distribution when compared to the corresponding distribution of poles.

Large differences between the distribution parameters of the poles and b-axes characterizes well defined (pure) clusters and girdles while similar values indicate clusters containing weak girdles or girdles containing weak clusters.

Limitation of the intersection angle for girdle distributions

Fine adjustments can be performed for improving the meaningfulness of the statistical results: the use of the weighting factor and/or different weighting mode, as the limitation of the intersection angle.

For the calculation of the b-intersections, it could not be appropriate to compare pairs of planes that are sub-parallel to each other and belonging to the same group of the closely related data. In the example used above of a cylindrically folded surface, computing planes from the two limbs of the fold and oriented far from each other will likely give more reliable b-axes than pairs of planes coming from the same limb and closely oriented. It can be therefore necessary to avoid computing intersections of the latter by setting a minimum angle (limiting angle) below which pairs of planes will not be considered. This angle can either be taken as the half of the confidence cone angle for the Poles distribution of set manually.

Application of weighting factor and weighting mode

As said above, the average orientation and statistical dispersion can take into account the weighting factor associated to each data. If the weighting flag is checked, the length of the unit vector (direction cosine) is simply multiplied by the weighting factor.

It will be seen elsewhere that the weighting factor itself can be applied in different way (this can be controlled in the Graphic Options dialog windows). In mode 1, the weight given to the data in the database is used as a multiplying factor. In mode 2, the square of the weight is used. In mode 3, the weight is used as the exponent on basis 10. Mode 1 is the simplest to use and has a little effect on the results. It is advised when no particular attention has been put foreword on this during the data collection. Mode 2 is convenient for fault-slip data and when the dimension of the measured fault plane has been expressed by one side of a planar square that fit the surface of the observed fault. Mode 3 is specially designed for earthquake fault-slip data, when the weight factor corresponds to the magnitude.

Distribution of linear data

For linear data, the procedure is exactly similar as with planar data. It uses directly the linear data instead as converting first the planar data into poles. At the end of the procedure, the results are converted back into linear data.

2/ Separation of data into subsets

The separation of a heterogeneous data set into homogeneous subset is largely based on the algorithm and computer code of Huang and Charlesworth (1989).

Separation method

With heterogeneous sets of orientation data, the mean orientation and dispersion of data cannot be estimated numerically using the mean resulting vector R, the Ficher concentration parameter KA or the Confidence Cone Angle as described by Fisher (1953), Fisher et al. (1987) and implemented by Dzik (1992) and Charlersworth et al. (1989). If data concentrations can be visually observed on stereographic projections they can be separated into homogeneous subsets using the numerical iterative sorting method based on dynamic cluster analysis developed by Hang and Charlersworth (1989).

This method needs a first estimate of the number of subsets and their orientation. After plotting all orientations on a lower hemisphere and a visual identification of the subsets, the estimated mean orientations of the subsets are given. These will serve as nucleus for attracting orientation data during the iterative separation process. The maximum number of valid subsets obtained after the separation will be constrained by the number of initial estimates. If a specified nucleus does not attract data during the procedure, the subset will be considered as nonexistent. Specifying more subsets than visually observed or specifying initial orientations incorrectly will not alter the separation results.

Using the initial nuclei, each orientation data will be assigned to the appropriate nucleus giving the smallest deviation angle. The mean vector of each nucleus will be computed using the orientations just assigned to them in order to obtain a better estimate of their mean orientation. This process is repeated until each orientation data is correctly assigned to a nucleus and the system stabilizes.

To this original method, we added a limiting cone angle to limit the attracting influence of the nuclei.

Implementation in the Statistics Module

In the Statistics panel of the Processing worksheet the drop-down lists below labels "Planes: Subsets" and "Lines: Subsets" control the number of subsets for separating heterogeneous data sets into subsets (Fig. X.1b.). Setting them to one will cause the computation of orientation statistics bases of single sets and determine their cluster or girdle distribution. Selecting more subsets (max. 5) will activate the separation method.

After specifying a number of subsets larger than 1, the stereographic projection will be refreshed, and a moving cross will appear at the end of the mouse arrow, together with a tiny small red circle representing the proposed orientation for the first nucleus and the a larger small circle of red dots representing the limiting cone angle. The current orientation of the pointer is displayed in the text box above the stereogram plunge and azimuth angles. The orientation of the first nucleus is fixed by pressing the right button on the mouse. The moving cross will again appear with the proposed second nucleus. Right clicking will fix it again and theses steps are repeated until all nuclei have been defined. The separation is performed and the results are displayed numerically on the text area above the stereogram with the subset number and associated symbol, the number of data attributed the estimated orientation of the nucleus, the calculated orientation, the Fisher concentration parameter KA and the mean cone angle MCA. The subsets are also displayed on the stereogram with different symbols for each

subset, a solid dot for non-attributed data, the computed orientations and the cone angle for each subsets (Fig. X.1.b).
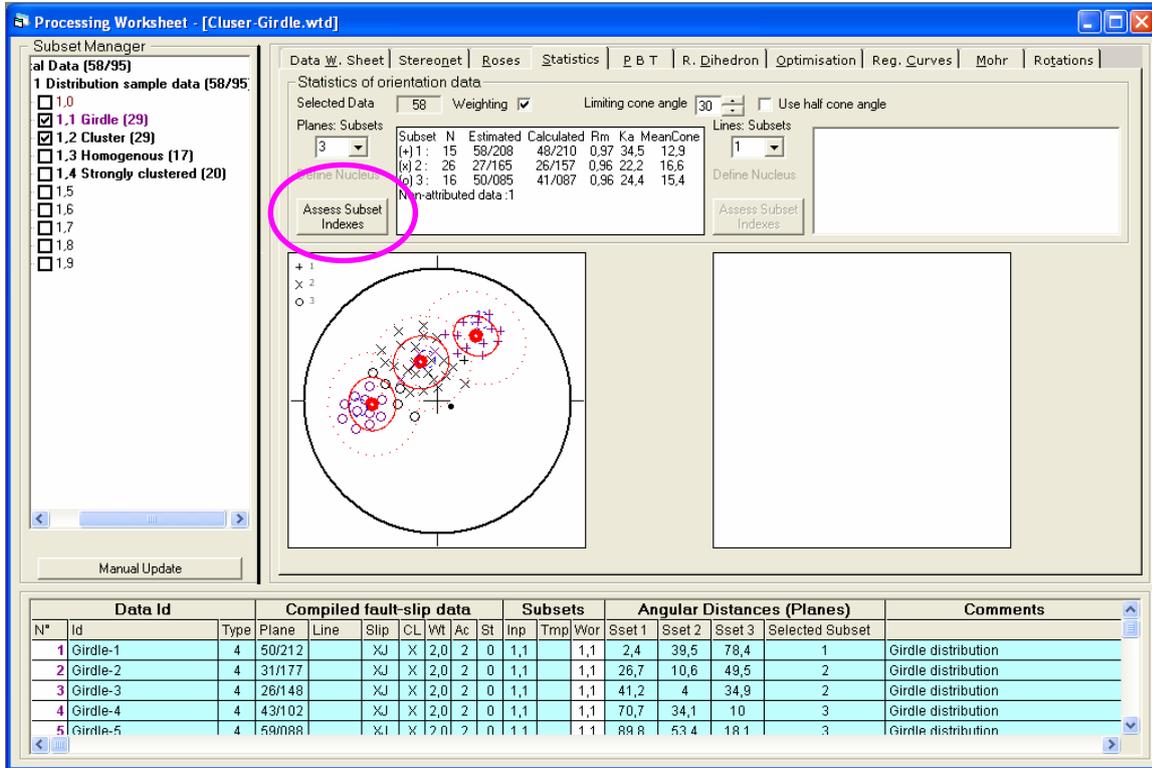

Figure X.1b: Statistics pane of the Processing Sheet, with orientation statistics for the subsets separated from the selected data. After separation into subsets, the button "Assess Subsets Indexes" is activated.

To illustrate the robustness of the separation method, for the same initial data sets, we defined a total of 4 nuclei, 3 of them at the vicinity of the identified subsets but not at their approximate center and the last in an area where no data exits. The process correctly separates the original data set into 3 valid subsets separation process results, almost identical to the ones obtained in the preceding run with only 3 initial nuclei. The forth nucleus have no data attributed and two data have not been attributed to any subset, being too far away from the initial nuclei taking into account a limiting cone angle of 30°.

Again, the process can be customized by adjusting the limiting angle and the weighting factor as for single datasets.

Working with Subset Indexes (reminder)

Before going further, it can be useful to remember the general principle of working with Subset Indexes (this is described in detail in the Data Subset Management document of the Win-Tensor User Guide).

For an efficient processing of structural data for statistical or tectonic stress analysis, it is convenient to group some selected data into subsets, according to their type (e.g. bedding planes, foliation, faults, fractures…), their clustering (this module) or their conformity to a particular stress tensor. For this purpose, a Subset Index has been introduced.

In Win-Tensor, the Subset Indexes are defined as an index associated to each data row in the data base, defining its belonging to a subset. The indexes are introduced initially in the Data Worksheet as Input Indexes. In the Processing Worksheet, the Subset Indexes they are transferred in the only editable column (with a white back color) and displayed as Working Indexes. The Subset Manager groups the data into subsets and controls the selection of subsets. The Subset indexes are composed of a 2-digit integer part (from 0 to 99) and a 1-digit decimal part (from 0 to 9) with a comma as separator. The integer part constitutes the Major (part of the) Index and the decimal part, the Minor (part of the) Index).

Assessing Subset Indexes to the separated data

After the iterative separation into homogeneous subsets, the angular distance between the data orientation and the calculated nuclei for each subset is displayed on the data table at the bottom of the Processing Worksheet, for every data row (Fig. X.2, columns labeled Nucl.1, Nucl.2, …). The orientation data has been assigned to a subset on the basis of the smallest angular distance between its orientation and the nuclei. This subset is identified by a number stored in column Assigned Subset.

Subset Index			Angular Distances (Planes)			
Input	Tmp	Working	Nucl.1	Nucl.2	Nucl.3	Assigned Subset
1,1		**1,1**	2,4	41,7	80,8	1
1,1		1,1	26,7	12,8	51,8	2
1,1		1,1	41,2	2	37,3	2
1,1		1,1	70,7	32,1	10,2	3

Figure X.2. Close up of the data table, zooming on the column displaying the angular distance between each data and all the calculated nuclei, with an additional column containing the index of the selected subset.

Once a satisfactory separation into subsets has been performed, the decimal part of the Subset Working Indexes (column with a white background) can be re-defined according to the separation performed. This can be done by calling the Assess to Subsets dialog box by clicking on button Assess Subset Indexes that has been activated after the separation (circled on Figure X.1a).

This dialog box (Fig. X.3a) displays drop-down lists in regard of the numbered subsets which define the decimal part of the subset indexes to which the separated data have to be attributed. Note that the range of application has to be defined before (selected/all Subsets and Data).
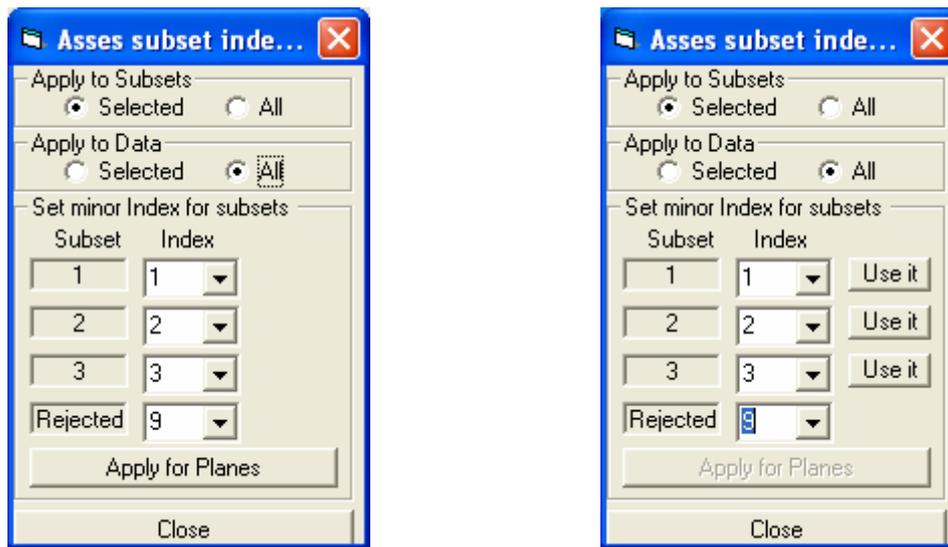
Figure X.3.a: dialog box Assess to Subset before application changing the subset indexes by pushing on button Apply. b: the same dialog box after assessing the indexes.

Clicking on Apply for Planes (or for Lines) will affect the minor part of the subset indexes of the selected data. This will cause also buttons labeled "Use it" to appear and will also desactivate the button Apply (Fig. X.3b). Pushing on one of these buttons "Use it" select the corresponding minor subset index (decimal part) of all major subset indexes (integer parts) and deselect the others, in order to activate only the data of the selected subset. Simultaneously, the drop-down lists within the current module are reset to 1, providing the distribution statistics for the selected subset. This can be repeated in order to get the statistics of all the separated subsets.

Care should be paid during this operation to avoid data already present in the minor subset indexes of some major indexes.
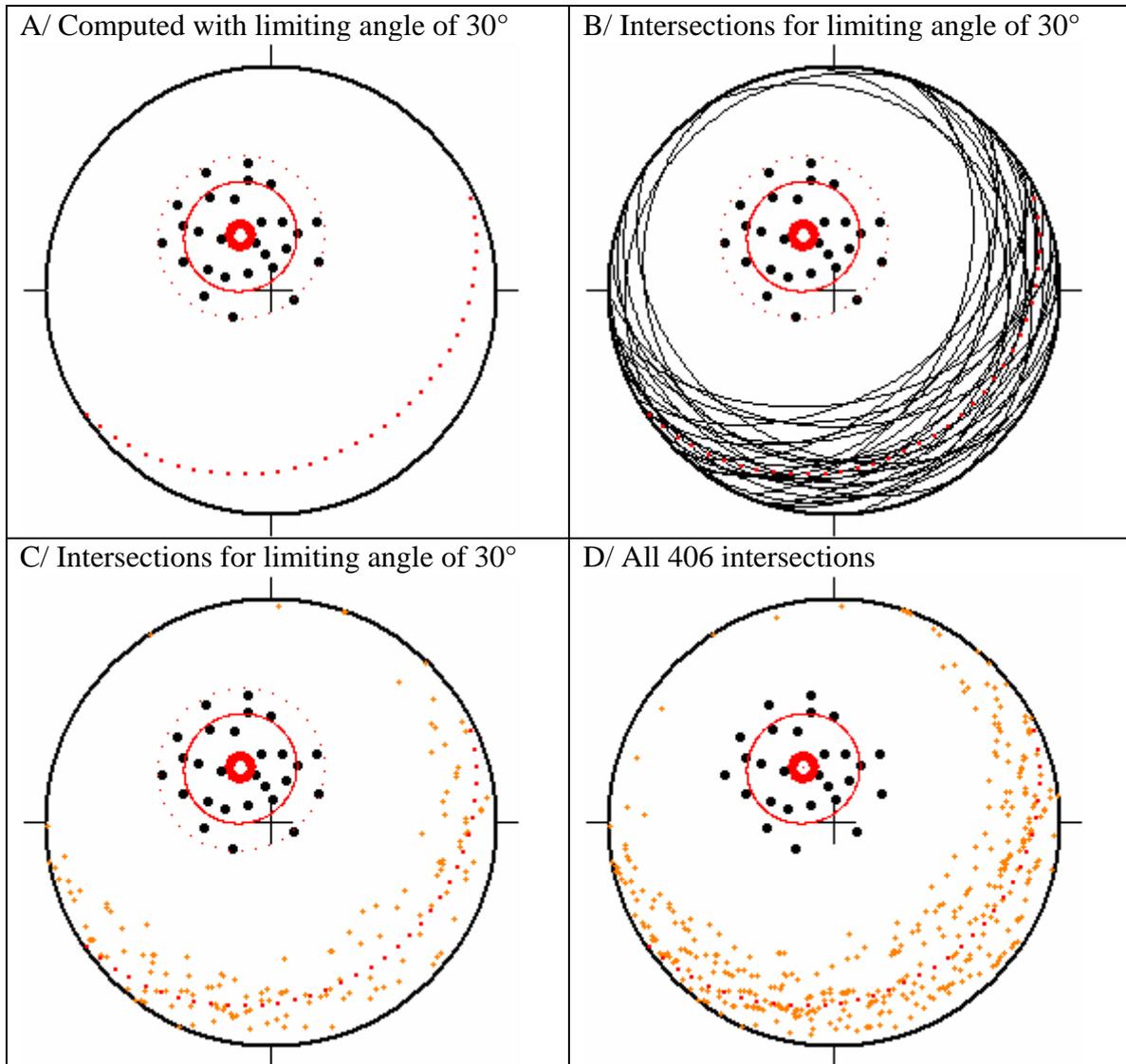
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Cluster distribution

Example X.1: orientation statistics of the cluster data subset.
A: 29 poles of planes (black), average orientation of poles (bold small red circle), Confidence Cone angle (20.2°, solid red small circle), limiting cone angle of 30° (small circle with thin red dots) and great circle normal to the average orientation (large red dots).
B: Similar as A, with planes represented by dark great circles.
C. Similar as A, with intersection plotted as small dots in orange. Note the good fit between the great circle and the girdle distribution of the intersections
D. Similar as D, with all 406 intersections (0°) limiting angle.

Limiting angle : 30°							Limiting angle : 0°						
Orientation statistics of Poles of Planes (n=29)							Orientation statistics of Poles of Planes (n=29)						
N	Data	Orient.	Rm	Ka	MCA	Distrib.	N	Data	Orient.	Rm	Ka	MCA	Distrib
29	Poles	23/151	0,94	15,7	20,2°	Cluster	29	Poles	23/151	0,94	15,7	20,2°	Cluster
153	b-axes	10/213	0,69	3,2	46,3°		406	b-axes	10/215	0,65	2,9	49,2°	

Girdle distribution

A/ Computed with limiting angle of 30° B/ Intersections for limiting angle of 30°

C/ Intersections for limiting angle of 30° D/ All 406 intersections

Example X.1: orientation statistics of the cluster data subset.
A: 29 poles of planes (purple), average orientation of b-axes (bold red triangle), Confidence Cone angle for b-axes (20.2°, solid red small circle), limiting cone angle of 30° (small circle with thin red dots), and great circle normal to average b-axis fitting the girdle distribution of poles (large red dots).
B: Similar as A, with planes represented by dark great circles.
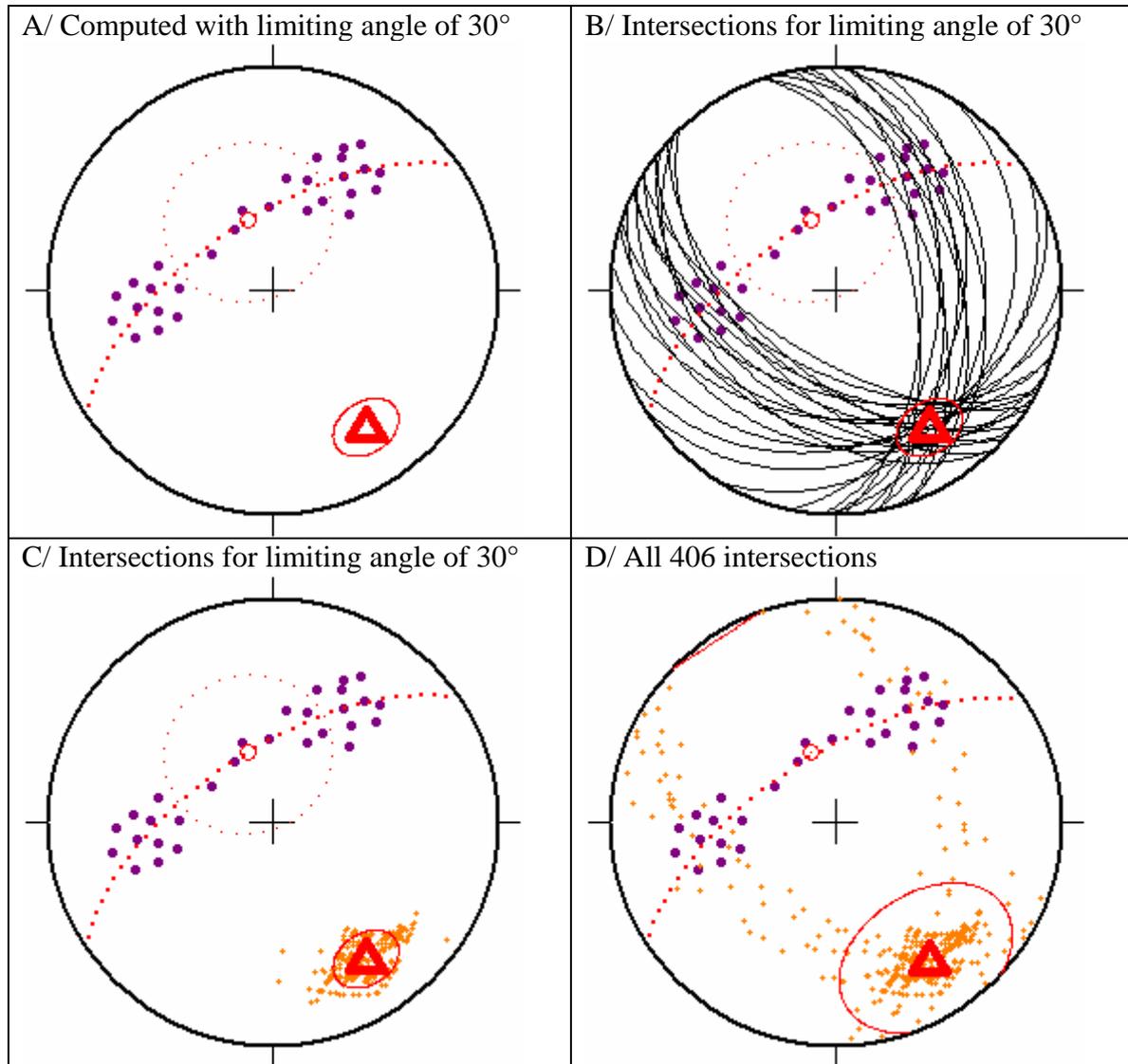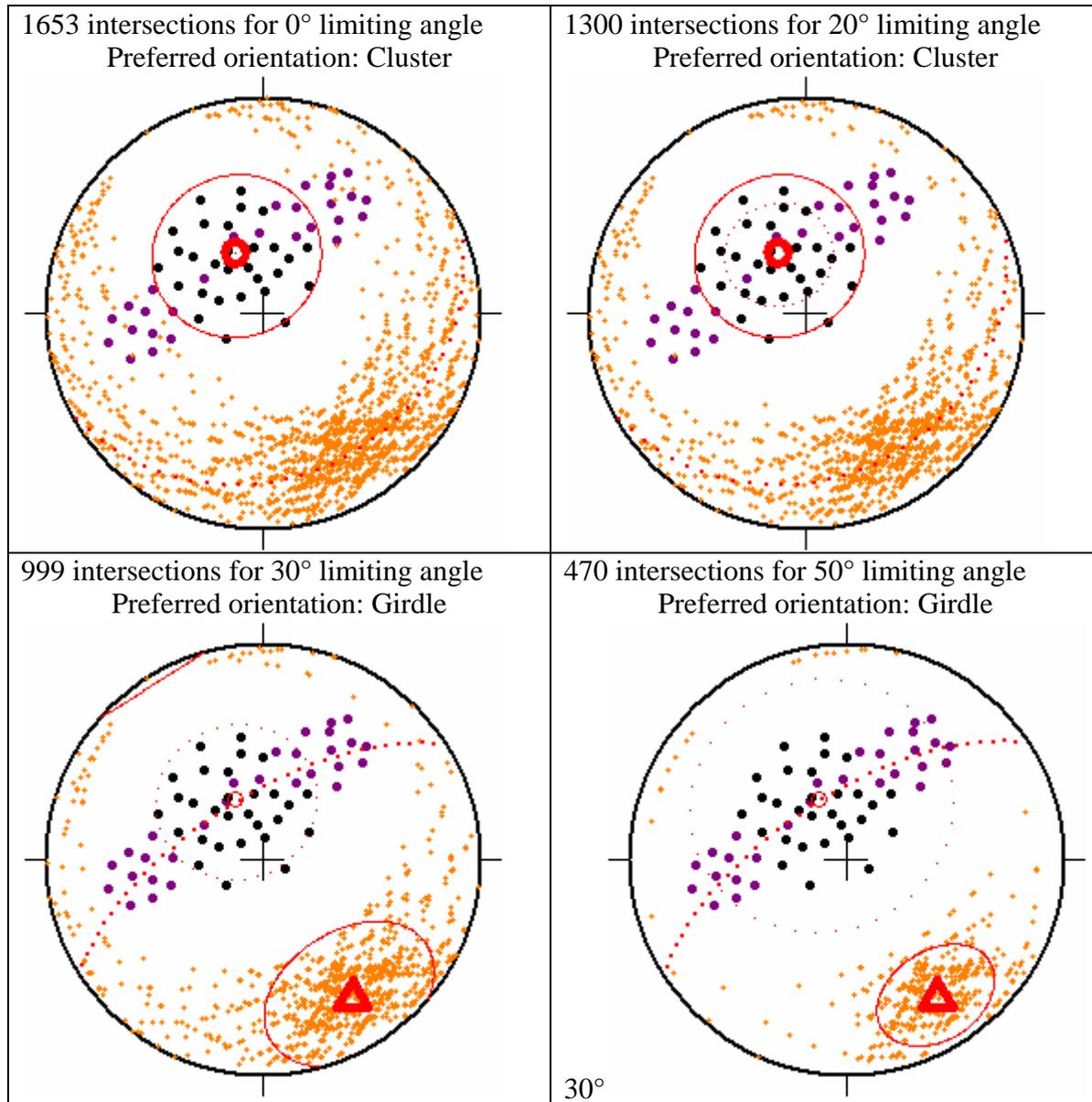C. Similar as A, with intersection plotted as small arrange dots. Note the good fit between the great circle and the girdle distribution of the intersections
D. Similar as D, with all 406 intersections (0°) limiting angle.

Limiting angle : 30°							Limiting angle : 0°						
Orientation statistics of Poles of Planes (n=29)							Orientation statistics of Poles of Planes (n=29)						
N	Data	Orient.	Rm	Ka	MCA	Distrib.	N	Data	Orient.	Rm	Ka	MCA	Distrib.
29	Poles	27/160	0,77	4,2	39,8°		29	Poles	27/160	0,77	4,2	39,8°	
238	b-axes	27/146	0,98	55,4	10,9°	Girdle	406	b-axes	25/147	0,88	8,1	28,7°	Girdle

Mixed distribution

1653 intersections for 0° limiting angle
Preferred orientation: Cluster

1300 intersections for 20° limiting angle
Preferred orientation: Cluster

999 intersections for 30° limiting angle
Preferred orientation: Girdle

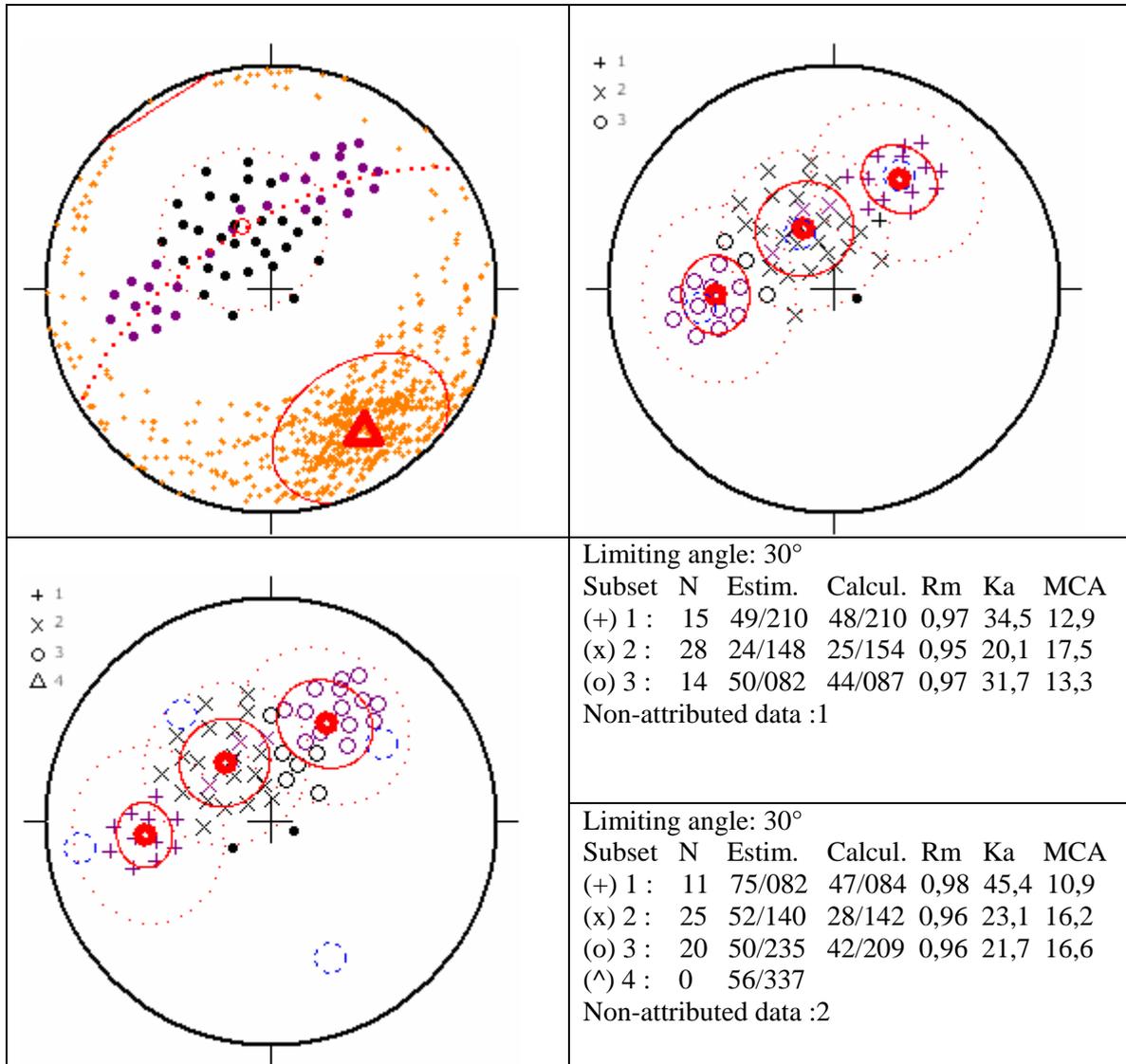470 intersections for 50° limiting angle
Preferred orientation: Girdle

30°

Example X.3: orientation statistics of a mixed cluster and girdle data subset.
Mixed data set formed by 29 poles of planes from the cluster data sets used above (black) and 29 poles from the girdle distribution (purple). Average orientations, Confidence Cone Angle, limiting cone angle and great circles as above.
This series of stereograms illustrate the effect of the progressive increase of the limiting angle between pairs of planes for computing the b-axes. The b-intersections are widely dispersed with no or limited angular limitation (limiting angle < 21°) and the poles are more clustered than the b-axes. With increasing limitation angle, the b-axes become progressively more clustered and their clustering surpasses the clustering of the poles. In consequence, the preferred distribution is cluster when the limiting angle is small, and girdle when it is high (more than 21° in the present case).

Separation into subsets

Limiting angle: 30°						
Subset	N	Estim.	Calcul.	Rm	Ka	MCA
(+) 1 :	15	49/210	48/210	0,97	34,5	12,9
(x) 2 :	28	24/148	25/154	0,95	20,1	17,5
(o) 3 :	14	50/082	44/087	0,97	31,7	13,3
Non-attributed data :1						

Limiting angle: 30°						
Subset	N	Estim.	Calcul.	Rm	Ka	MCA
(+) 1 :	11	75/082	47/084	0,98	45,4	10,9
(x) 2 :	25	52/140	28/142	0,96	23,1	16,2
(o) 3 :	20	50/235	42/209	0,96	21,7	16,6
(^) 4 :	0	56/337				
Non-attributed data :2						

Example X.4: Orientation statistics of the subsets separated from the mixed cluster and girdle data subset used in Example X.3.

Mixed data set formed by 29 poles of planes from the cluster data sets used above (black) and 29 poles from the girdle distribution (purple). Intersection of great circles plotted as small dots in orange. Average orientations, Confidence Cone Angle, limiting cone angle and great circles as above. Small blue dotted circles: estimated orientations of the nuclei.