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Optimization Functions for Stress tensor optimization

Parameters of the stress tensor

S1mag: magnitude of principal maximal stress axis (sigma1)
S3mag: magnitude of principal minimum stress axis (sigma3)
Psi = S3mag / S1mag

The magnitude of σ_1 (S1mag) is set to an arbitrary value (we use here S1mag = 100).
The Magnitude of σ_3 (S3mag) is obtained using the ratio Psi:
S3mag = S1mag * Psi

Resolved stress on a plane after application of the stress tensor

Nmag: magnitude of resolved normal stress on the plane
Tmag: magnitude of resolved shear stress on the plane
Phi: friction coefficient = arctan (Tmag / Nmag)
Alpha : angle between observed slip line and resolved shear stress direction on the plane

Optimization functions

F1 : Slip-shear misfit angle (slip deviation) to be minimised
F1 = Alpha

F2 : Square of slip-shear misfit angle slip deviation) to be minimised
F2 = (Sin(Alpha / 2) ^ 2) * 360 (Angelier, 1997, 1991)

F3 :Friction angle to be maximized
F3 = Phi

F4: Resolved shear stress to be maximized
F4 = Tmag

F6: Friction angle to be minimised
F6 = Phi

F5: Composite function to be minimised
See details hereafter

F7: Resolved shear stress to be minimised
F7 = Tmag

F8 : Resolved normal stress to be minimised
F8 = Nmag

F9 : Resolved normal stress to be maximized
F9 = Nmag

Function F5

In the Rotational Optimization module, an initial reduced stress tensor is progressively adjusted to the data set by a procedure involving an iterative testing a number of stress tensors by computing the resolved stress on the fault planes and determining an optimization function, in order to find the minimum for this function for all possible configurations of the 4 parameters of the reduced stress tensor. As the minimum for this function is not determined by a least square method, the form of the function can be complex, which constituted an advantage that is largely exploited in Win-Tensor.

Testing a tensor on a fault-slip data is done by computing the two components of the resolved stress on the fault plane: the shear stress and the normal stress. Both are orientation vectors with a direction and a magnitude. The resolved shear stress describes the orientation of the resolved slip on the plane and the associated sense of slip. The pointing direction of the resolved shear vectors depends from the sense of the dip-slip component, by convention pointing downward for normal sense and upward for reverse sense. The orientation of the resolved shear stress can be compared with the orientation of the slip line in the case of slickensided faults, and its pointing direction, to the observed slip sense, and expressed as a vectorial angle (the slip deviation alpha). The magnitudes of the normal and shear components of the resolved stress can be used for estimating the ability of the plane to slip (for shear faults or fractures) or not to slip (for tension and compression joints).

All these characteristics are exploited by the optimization function F5 which consists of two terms. The first term exploits the directional part of the resolved shear stress (orientation and sense) for minimizing a misfit angle and the second term exploits the resolved magnitudes for optimizing the resolved normal and shear stresses magnitudes to favor slip on the plane. The value of F5 is the sum of these two terms, their relative contribution defined by a factor Cratio ranging between 0 (use of first term only) and 1 (use of second term only). The general form is as follows:

F5 = [Term 1 * (1-C.ratio)] + [Term 2 * (Cratio)]
With Cratio = 0 to 1

F5 is further normalized to constrain its value between 0 and a maximum fixed by F5max. The default value of F5max is set to 360. The exponent E which appears in the function is set by default to 2. The values of Cr, F5max and E are set in the General Options and can be modified when working with a data set.

Significance of Shear Fractures in stress inversion

In addition to slickensided fault planes, shear fractures can bear also information that can be used in stress inversion. Function F5, in addition to favor slip on the plane, can also use the apparent slip sense in order to reject incompatible solutions that model an opposite slip sense than the observed apparent sense.

A shear fracture is a fracture plane for which there are reasonable indications that it has been activated by slip, but where no indications for the slip direction can be seen. Some of them crosscut reference planes that have been displaced at their contact. In function of the inclination of the observation surface, an apparent dip-slip movement (normal or reverse) can be defined for subvertical observation surfaces and an apparent strike-slip movement (dextral or sinistral) can be defined for subhorizontal observation surfaces. The differentiation between dip-slip and strike-slip faults is conveniently made by reference to the inclination of the associated b kinematic axes (≤ 45° for dip-slip and > 45° for strike-slip).

The composite function F5 for such shear fractures has an additional term that allows using the information related to he fact that the fracture was activated as a slip plane, and also that slip occurred in a particular apparent sense (if observed). Because no slip line is recorded, the slip deviation alpha cannot be accessed, but the sense of the resolved shear stress on the plane (an oriented vector) can be compared with the apparent slip sense to estimate his compatibility with the stress tensor, regardless to the direction of resolved shear. The shear fracture is considered compatible if both slip and shear senses are similar in a vectorial format. If both senses are opposing each other, then the shear fracture is considered incompatible.

Function F5, a composite function

Function F5 which is aimed to be minimised, can be customized by acting on a number of parameters and its application depends on the type of fault-slip data under consideration:

- For slickensided fault planes, it primarily minimises the slip deviation between the observed and resolved slip vectors on the plane and in addition, maximizes of resolved shear stress magnitude and minimises the resolved normal stress magnitude in order to favor slip on the plane.
- For shear fractures (fractures for which shear movement is suspected but no indicators for slip direction have been recorded), F5 only maximizes of resolved shear stress magnitude and minimises the resolved normal stress magnitude in order to favor slip on the plane.
- For shear fractures with apparent sense of movement F5, in complement to favor slip on the plane as for a simple shear fracture, also minimises a misfit angle between the shear plane and the orientation of principal stress axis $\sigma 1$.
- For both tension fractures (plume joints without fringe zone, tension gashes, mineralized veins, magmatic dykes...) and compression fractures (sylolite planes, pure shear cleavage planes), F5 minimises the resolved shear stress in order to prevent slip on the plane. For tension fractures, it also minimises the resolved normal stress while for compression fractures, it maximizes the resolved normal shear stress.

Normalisation of the resolved shear and normal stress magnitudes

Because in tectonic stress inversion, the resolved normal and shear stresses on a plane depend only on the 4 parameters of the reduced stress tensor, Function F5 has to be insensitive to changes in extreme principal stress magnitudes as defined by ratio Psi = $\sigma 3/\sigma 1$ and by the absolute magnitudes (governed by the value of σ1, Psi and R). The resolved shear and normal stress magnitudes have therefore to be normalized. In Win-Tensor, we pose arbitrarily as normalized conditions: S1mag = 100 and S3mag = 0 (Psi = 0).

Posing the difference between S1mag and S3mag as:

ΔNmag = S1mag - S3mag   (S3mag = S1mag * Psi)

, the magnitudes of the resolved stresses normalized for S1mag and S3mag are expressed by:

Nnorm = 100 – (S1mag – Nmag) * (100 / ΔNmag)
with Nnorm = Nmag for Psi = 0 and S1mag = 100

Tnorm = Tmag * (100 / ΔNmag)
with Tnorm = Tmag for Psi = 0 and S1mag = 100

The magnitude of the resolved normal and shear stresses on a plane are conveniently expressed on a Mohr circle for resolved stresses, with the normal stress as the Y axis and the shear stress as the X axis. When normalized as above, the normal stress range from 0 (the normalized magnitude of σ3) to 100 (the normalized magnitude of σ1) and the shear stress, from 0 to 50 (half of the difference between the normalized magnitudes of σ1 and σ3, or ΔNmag/2).

A particular property of the relation between the magnitudes of σ1 and σ3 on a Mohr circle, which will be used in the Function F5, is the maximum possible value for the sum Nmag + Tmag (NTmax).

On a Mohr circle centered on C (ΔNmag/2 on the Y axis), we define the angle *beta* as the vectorial angle between the horizontal vector originating on C and pointing to S3mag, and the vector originating on C and ending at a point on a Morh circle defined by Nmag and Tmag. For all points on the external Mohr circle and with reference to the angle beta, we get:

Nmag = (ΔNmag / 2) – (ΔNmag/2 * cos(beta))
Tmag = (ΔNmag / 2) * sin(beta)

With optimally oriented tension joints, beta = 0° (both null shear and normal stresses) and, conversely, for the worse oriented tension joints (maximum value of shear and normal stress), beta is 135° (Fig. X.1).

With optimally oriented compression joints, beta = 180° (null shear stress and maximum normal stresses) and, conversely, for the worse oriented tension joints (minimum combination of shear and normal stress), beta is 45° (Fig. X.1).

In both cases, NTmax = (Nmag + Tmag) = 120,7106781, for S1mag = 100 and S3mag = 0

Rem: in version 2.1.4 and older of WinTensor, the value of NTmax was set approximately to 122.84 for tension joints and to 120.44 for extensional joints, giving respectively F5max values of 353.8 and 361.62

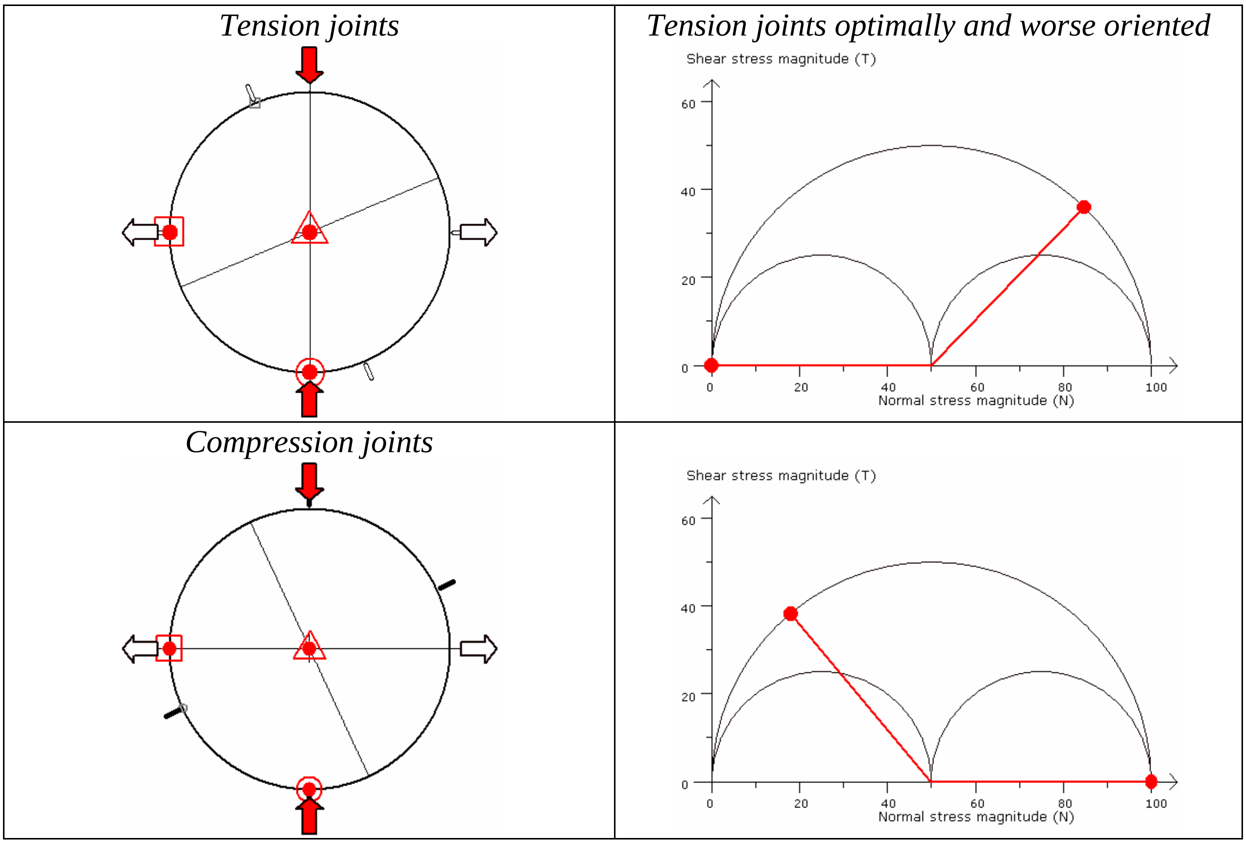

Figure: X.1 Tension joints (above) and Compression joints (below), optimally oriented (respectively perpendicular to s3 and to s1), or worsely oriented oblique to both $\sigma 1$ and $\sigma 3$ in a strike-slip stress field with both $\sigma 1$ and $\sigma 3$ horizontal, $\sigma 1$ N-S, $\sigma 3$ E-W and R = 0.5. Left: lower hemisphere stereographic projection, right: Mohr diagrams with position of the related data (red dots) as function of their magnitudes of resolved shear and normal stress and angles beta.

<u>Minimizing the misfit angle (first term)</u>

The first term of the function F5 minimises a misfit angle which is related to the directional part of the resolved shear stress (orientation and sense). It will be expressed in a different way for slickensided faults and for shear fractures without slip lines.

As proposed by Angelier (1977; 1991), the misfit angles are minimised using their sinus in a function of the form:

F = (sin (Misfit / 2) ^ E) * F5max

With, E the exponent and Fmax, the maximum value of the function (by default, E = 2 and F5max = 360).

For faults with slip lines, the Misfit factor to minimise in the first term of the function is expressed by the slip deviation Alpha between the observed slip line and the resolved shear stress on the plane.

Both observed slip line and the resolved shear stress are expressed by oriented vectors, pointing downwards for slip planes with a normal did-slip component of movement and upward for reverse dip-slip component. The slip deviation Alpha is therefore a vectorial angle ranging from 0° (both slip and resolved shear vectors parallel and pointing to the same direction), to 180° (both slip and resolved shear vectors parallel and pointing in opposing directions) and with 90° as the largest angular misfit (the two vectors at right to each other) when the observed slip sense is unknown.

We get, for known sense of movement:

Misfit = Alpha

and for unknown sense of movement:

Misfit = Alpha (Alpha > =90°)
Misfit = 180 - Alpha (Alpha > 90°)

For shear fractures (slip plane but without indication of slip direction), the first term of the function F5 favors slip on the plane by minimizing a misfit angle computed in function of the angular relation between the slip plane and the principal stress axis $\sigma 1$.

The definition of this Misfit factor relies on the basic rock mechanic principle following which when a new (neoformed) shear fracture develops in an intact rock under applied stress, the optimal half-dieder angle between the fracture plane and $\sigma 1$ is close to 30°. Defining *gamma* as the angle between the fracture plane and the maximal principal stress axis ($\sigma 1$), the misfit "angle" is constructed in order to be to equal to 0 when $\sigma 1$ is at 30° from the plane, and to increase to a maximum value of 30 towards parallelism with the shear plane and, in the opposing sense, to 60 when $\sigma 1$ is orthogonal to the plane:

Misfit = 30 - *gamma* for *gamma* < 30°
Misfit = *gamma* - 30 for *gamma* > 30°

For reactivated faults (faults already ruptured by an earlier event or other discontinuities), this half-dieder angle is set to 45°, the position which gives the largest resolved shear stress:

Misfit = 45 - *gamma* for *gamma* < 45°
Misfit = *gamma* - 45 for *gamma* > 45°

When the apparent slip sense is known thanks to the observed displacement of cross-cutting features, an additional condition is to be met: the sense of the resolved shear stress on the plane should be compatible with the observed apparent slip sense. For compatible situations, we keep the same definition of the Misfit factor as above. For **non-compatible** slip sense, the misfit value is constructed in order to increase from a value of 120 when $\sigma 1$ is parallel to the plane, to 180 when $\sigma 1$ is at 60°, then to decrease to 150 when $\sigma 1$ is orthogonal to the plane:

Misfit = gamma + 120 for *gamma* < 30°
Misfit = 180 - (gamma - 60) for *gamma* > 30°

Smoothing

The Misfit factor calculated for shear joints as above induces a progression in the misfit values which particularly suitable when optimizing a stress tensor by the rotational iteration process. The sharpest difference in value at the transition between compatible and non-compatible fractures in term of apparent slip sense is from 30° (45°) and 120°.

Another option would be to keep this Misfit factor as null (0°) for all situations where the apparent slip is compatible with the modeled one and high (360°) for incompatible slip senses. This will create sharp boundaries between compatible and non-compatible data which can be useful to reject incompatible data but will not help much in the tensor optimization.

In Win-Tensor, it is possible to choose between the two by checking the "Smoothing" box for a progressive evolution from compatible to non-compatible situation as in the first case or un-checking it for sharp contrasts between compatible and non-compatible situations as in the second case.

<u>Optimizing the resolved stress magnitudes (second term)</u>

For both tension and compression joints, their optimal orientation with respect to the external stress field should be such as it generates no resolved shear stress, together with the minimum possible resolved normal stress (for extension joints) or the maximum possible resolved normal stress (for compression joints).

For tension joints, function F5 minimises both normalized normal stress and shear stress magnitudes and is proportional to the sum of the normalized normal and shear stress magnitudes elevated to its square:

$F5_{TJ}$ = ((Nnorm + Tnorm) ^E) * (F5max / (NTmax^E))
with (0 <= F5 <= F5max)

For compression joints, function F5 maximizes then normalized normal stress magnitude and minimises the normalized shear stress magnitude. It is proportional to the sum of the difference between the magnitude of Sigma 1 and normalized normal stress magnitude, and the normalized shear stress magnitudes elevated to its exponent:

$F5_{CJ}$ = (((100 - Nnorm) + Tnorm) ^E) * (F5max / NTmax^E)
with (0 <= F5 <= F5max)

In both cases, the term F5max/NTmax is used to bring the value of F5 within the range 0 - F5max (with F5max fixed at 360).

When the first term of the function is elevated to its square, NTmax has also to be elevated to its square in order to reach the same F5max value.

For shear fractures without apparent slip sense indication (without slip lines), function F5 minimises the normalized normal stress magnitude and maximizes the normalized shear stress magnitude in order to reduce the friction on the plane and to favor slip. It is proportional to the sum of the normalized normal and stress magnitudes and the difference between the normalized shear stress magnitude from its maximum possible value:

$F5_{MJ}$ = (((Tcomp + Nnorm) – ΔNmag /2) + 20,7106781) ^E) * (F5max / (NTmax^E)

With Tcomp expressing the difference between the actual value of Tnorm and its maximum possible value (complementary value of Tnorm):

Tcomp = (ΔNmag / 2) – Tnorm

Simplifying, we get:

$F5_{MJ}$ = ((Nnorm - Tnorm + 20,7106781) ^2) * (F5max / (NTmax^E)

The term ΔNmag / 2 ensures that the value of F5 do not change with the stress ratio phi. The addition of 20,7106781 (equivalent to NTmax – ΔNmag) ensures that the value of the F5 remains strictly positive and is null for the optimal orientation of the shear joint in terms of slip activation.

In order to bring the value of F5 with the range (1 - F5max), the first term has to be multiplied by (F5max / 120.7106781)

As the first term is elevated to its square, 115 has also to be elevated to its square in order to reach the same F5max value.

With optimally oriented shear joints, beta = 45° (best combination of shear stress and normal stress for fault failure) and, conversely, for the worse oriented shear joints (minimum combination of shear and normal stress), beta is 180° and NTmax = ((S1mag-Nnorm) + Tnorm) = 120,7106781.

Rem: in version 2.1.4 and older of WinTensor, the value of NTmax was set approximately to 115 for shear joints and the additional parameter to 20.47, giving the *maximum possible value for F5SJ of 392.0*

<u>Combining the two terms in F5</u>

The final value of the function F5 is computed with the Misfit value instead of the deviation angle alpha:

F5ShearFract = [(sin(Misfit/2) ^E) * F5max * (1-R)] + [$F5_{SJ}$* (R)]

The relative weight of the two terms of the function F5 can be adapted by changing the value of R. For slickensided faults, R is kept small (20/360 = 0.05) in order to give a large weight of the first term (max. value = 342) and a small weight of the second term (max. value = 18).

Rem: in WinTensor, up to version 2.1.4, the value of R was fixed in order that the first maximum value of the first term as 340 and of the second term, 20 (R = 0,05555). In the following versions, this parameter can be adapted in the General Options to allow the user to vary the relative contributions of these two terms in order to find the optimal way. Up to now, no tests have been made to check the impact of this parameter, but for the analysis of fault-slip data, the minimization of the slip deviation alpha has to remain dominant.

Customizing the function F5

Function F5 can be customized by acting on a number of parameters:

Weighting: Controls to use of the weighting factor of individual data during the processing. When Weighting is turned off, the weight of all data is set to 1 and when it is turned on, weighting is done according to the Weighting mode (see hereafter). Weighting can be turned on or off by checking or un-checking the Weighting box which is displayed in all processing panels. Its default value is defined in the Graphic Option dialog box (Menu / Graphic / Graphic Options).

Weighting mode: Controls the way the weighting factor is used in computing the average value of the Misfit parameter and of the Optimization function. Its value is defined in the drop-down list next to the Weighting check-box in the Graphic Option dialog box. The value of the optimization function F5(i) of individual fault-slip data (i) is affected by the corresponding weighting factor w(i) to derive the weighted function F5w(i) in the following way:

- Mode 1: F5w(i) = F5(i) * w(i)
- Mode 2: F5w(i) = F5(i) * w(i)^2
- Mode 3: F5w(i) = F5(i) * 10^w(i)

Mode 1 is the usual way of working in Tensor until now. Modes 2 and 3 have recently been added. Mode 2 can be used with fault-slip data in order to take into account the surface area of the fault, expressed as a square with side length measured as w(i). Mode 3 can be used for focal mechanisms when the magnitudes are stored in w(i). In mode 3 however, the final result will be dominated by the largest magnitude focal mechanism. It should be remember that the scale and units of w(i) are arbitrary but should be defined in a consistent way for the entire data set.

Smoothing: As explained above, only applies to the first term of the shear fractures with apparent sense of slip.

Cratio: Controls the relative contribution of the two terms in function F5 as detailed above. Its default value is defined in the General Options (Menu / Tools / Options) and it can be modified in the Optimization panel (upper part of main frame). In general, this ratio is kept low (0.05) in order to give more importance to the first term of F5 which relies on the orientation component of the resolved shear stress (for faults) and on the relationship between the fracture plane and σ1 (for shear fractures). Setting Cratio to 1 will deactivate completely the first term of F5 and the minisation will be based only on the normalized magnitudes of the resolved stresses.

Exponent: Controls the value of the exponent E in the first term of the function F5. Its default value is defined in the General Options and it can be modified in the Optimization panel (upper part of main frame).

F5Max: Controls the maximum possible value of F5 for the worst-fit data. Its default value is defined in the General Options. This does not affect the processing and separation results.

Using slip sense: Defines weather the observed slip sense of slickensided faults has to be used for optimizing the stress tensor. By default, the observed slip sense of the measured fault slip data is used as an additional constraint in the tensor optimization. With the checkbox “Use Slip Sense” in the upper part of main frame, it can be decided not to use the observed slip sense. This will restrain the maximum value for the Slip Deviation angle to 90° instead of 180° in the first term of F5.

Using slip sense for particular Confidence factor: Details the use of the observed slip senses in function of the Confidence factor. In the dialog window “Process fault-slip data and separate into Subsets” (Menu / Data / Process & Separate fault-slip data), the use of the observed slip sense can be further refined in function of the Confidence factor for the slip sense determination (frame Use Slip Sense). Option “Use All Slip Sense” in this frame is coupled to the checkbox “Use Slip Sense” in the Optimisation pane. When it is unchecked, the option “Use only slip senses for” is activated in the dialog window and it is possible to determine for which Confidence factor this applies (Certain, Possible and and/or Supposed). It can be a good practice to deactivate the use of slip sense for data with a Supposed Confidence factor and to activate the two others. This will avoid inappropriate data separation as errors in the slip sense determination can be frequent in the Supposed Confidence category.

Faults as Shear fracture: Turns slickensided faults into shear fracture with apparent sense of movement for testing purpose. The slickensided fault data can be turned into fracture planes with apparent sense of movement (NF, IF, DF or SF), without affecting the data base. This can be activated using the check box “Fault as N, I, D, S fracture“, located above the stereonet.

Faults as MJ: Turns slickensided faults into shear fracture without apparent sense of movement for testing purpose. When the slickensided fault data have been be turned into fracture planes with apparent sense of movement, it is further possible to suppress the information on the apparent sense of movement, turning the faults as shear fracture. When the “Fault as N, I, D, S fracture“ box is checked, an additional check box “as MJ” appears next to it. Checking it will activate this option.

References:

Angelier, J. (1977). La reconstruction dynamique et géométrique de la tectonique de faille à partir des mesures locales (plans de faille, stries, sens de jeu, rejets): quelques précisions. Compte Rendus de l'Académie des Sciences de Paris 285(D), 637-640).

Angelier, J. (1991). Inversion directe et recherche 4-D : comparaison physique et mathématique de deux modes de détermination des tenseurs des paléocontraintes en tectonique de faille. Compte Rendus de l'Académie des Sciences de Paris 312(II), 1213-1218).